DVCS/DVMP fits in QCD

Dieter Müller

GPD representations

DVMP in the collinear factorization framework

Other frameworks

NLO corrections

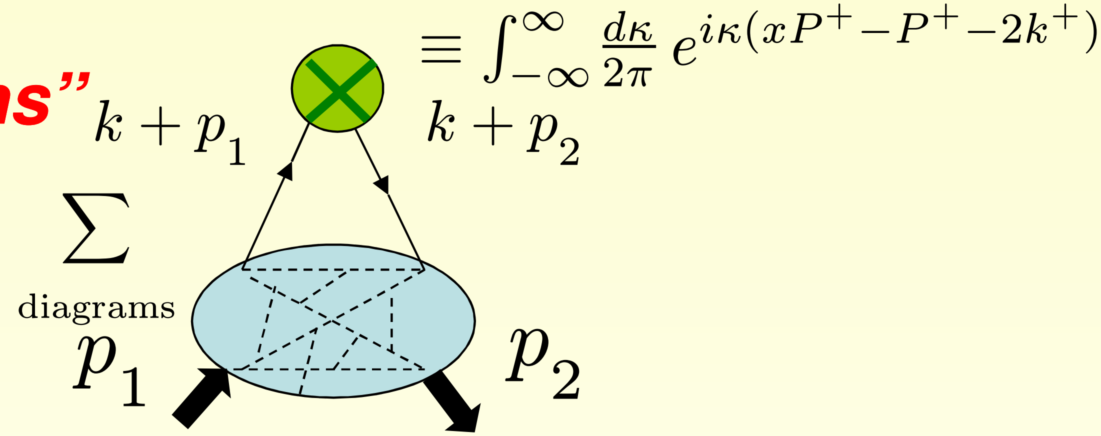
DVCS+DVMP small x_B fits at LO

in collaboration with

M. Meškauskas, K. Passek-Kumerićki, T. Lautenschlager, A. Schäfer
K. Kumerićki

C. Bechler and D.M., arXiv:0906.2571
GPD description of π+ electroproduction from HERMES/JLAB
M. Meśkauskas and D.M., arXiv:1112.2597
flexible GPD model fits@LO for small *x* and fits to vector mesons/DVCS H1/ZEUS data
DM, T. Lautenschlager, K.Passek-Kumerićki, A. Schäfer, arXiv:1310.5394
NLO corrections to DVMP in conformal space
T. Lautenschlager, DM, A. Schäfer, to be submitted
fits to H1/ZEUS data

Overview: GPD representations



``light-ray spectral functions”

diagrammatic α-representation

DM, Robaschik, Geyer, Dittes, Hořejši (88 (92) 94)

called ***double distributions***

A. Radyushkin (96)

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann, Jakob, Kroll (98,00)

Diehl, Brodsky, Hwang (00)

SL(2,R) (conformal) expansion

(series of local operators)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05); ….

one version is called Shuvaev transformation, used in `dual’ (*t*-channel) GPD parameterization

Shuvaev (99,02); Noritzsch (00) Polyakov (02,07)

each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM 07])**

Summing up conformal PWs

- GPD support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

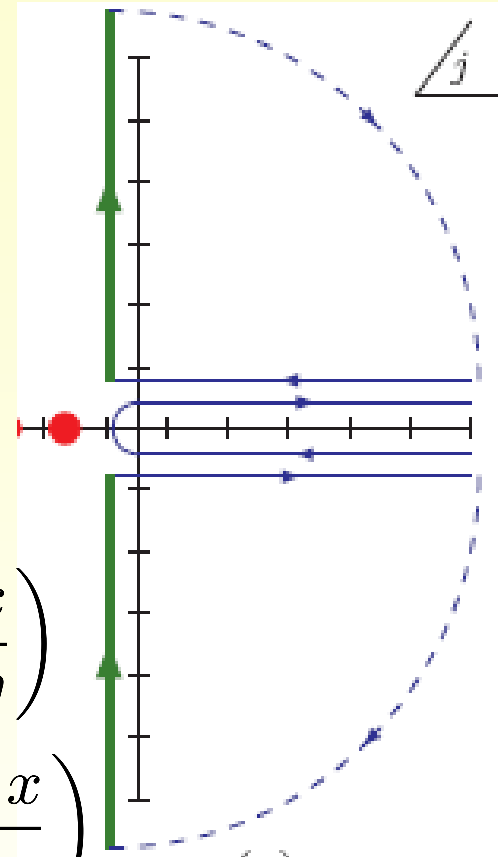
$$\mu \frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\ p_j(x,\eta) \propto \theta(|x| \le \eta)\frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:
  - smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
  - mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
  - dual parameterization [M. Polyakov, A. Shuvaev (02)]
  - based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
  - ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Sommerfeld-Watson transform



- ✓ rewrite sum as an integral around the real axis:

$$F(x,\eta,\Delta^2) = \frac{1}{2i}\oint_{(0)}^{(\infty)} dj\,\frac{1}{\sin(\pi j)} p_j(x,\eta)\,F_j(\eta,\Delta^2)$$

- ✓ find appropriate analytic continuation of p_j and F_j (Carlson's theorem)

$$p_j(x,\eta) = \theta(\eta-|x|)\eta^{-j-1}\mathcal{P}_j\left(\frac{x}{\eta}\right) + \theta(x-\eta)\eta^{-j-1}\mathcal{Q}_j\left(\frac{x}{\eta}\right)$$

$$\mathcal{P}_j(x) = \frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(1/2)\Gamma(1+j)}(1+x)\,{}_2F_1\left(\begin{matrix}-j-1,\, j+2\\ 2\end{matrix}\middle|\frac{1+x}{2}\right)$$

$$\mathcal{Q}_j(x) = -\frac{\sin(\pi j)}{\pi}\,x^{-j-1}\,{}_2F_1\left(\begin{matrix}(j+1)/2,\,(j+2)/2\\ 5/2+j\end{matrix}\middle|\frac{1}{x^2}\right)$$

- ✓ change integration path so that singularities remain on the l.h.s.

$$F(x,\eta,\Delta^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\,\frac{1}{\sin(\pi j)} p_j(x,\eta)\,F_j(\eta,\Delta^2)$$

Model based on SL(2,R) and SO(3) PWE

- SL(2,R) GPD moments: $F_j(\eta,t) = \sum_{J=J^{\min}}^{j+1} f_j^J(t)\, \eta^{j+1-J}\, \hat{d}_J(\eta)$

  partial wave amplitudes depending on j and J   *reduced Wigner rotation matrices*

- taking 2 better 3 SO(3) PWs:
  (two parameters s_2 and s_4)

$$f_j^{j-1}(t) = s_2 f_j^{j+1}(t)\,,$$
$$f_j^{j-3}(t) = s_4 f_j^{j+1}(t)\,,$$

- resulting CFF easy to handle:

$$\mathcal{F} = \frac{1}{2i} \sum_{\substack{k=0\\ \text{even}}}^{4} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \frac{2^{j+1+k}\Gamma(5/2+j+k)}{\Gamma(3/2)\Gamma(3+j+k)} \left(i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right)$$
$$\times s_k E_{j+k}(\mathcal{Q}^2) f_j^{j+1}(t) \hat{d}_j(\xi), \quad s_0 = 1$$

- zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha' t} \left(1 - \frac{t}{M_j^2}\right)^{-p}$

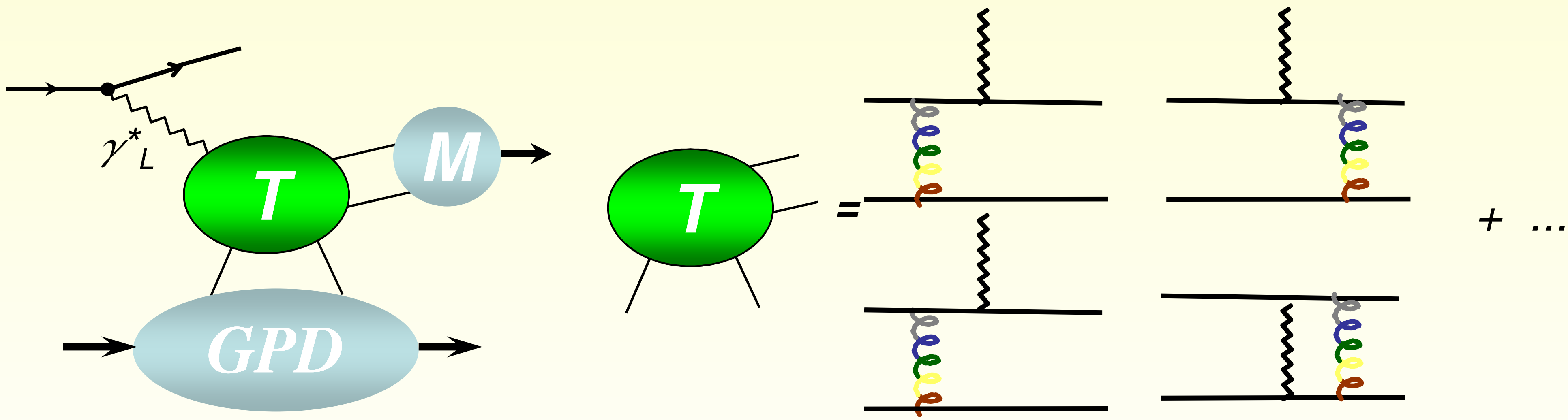
  PDF   *\`pomeron intercept' (biuld in PDF) + Regge slope*   *residual t dependence*

2x(2, 3, or 4) parameters:

***s_2, s_4, M or b*, (perhaps α')**

Collinear factorization framework

at large Q^2 **longitudinal** DVMP amplitude factorizes at twist-two level into a hard scattering part, GPD, and a meson distribution amplitude (DA) **[Collins, Frankfurt, Strikman 98]**, e.g.,



collinear factorization = integrating out k_T

perturbative + non-perturbative corrections

$$\mathcal{F}(x_{\rm B},t,\mathcal{Q}^2) = \frac{C_F f_{\rm M}}{N_c \mathcal{Q}} F(x,\xi,t,\mu_{\rm F}^2) \overset{x}{\otimes} T\left(\frac{\xi-x}{2\xi},\bar{v}\middle|\cdots\right) \overset{v}{\otimes} \varphi(v,\mu_\varphi^2)\Big|_{\xi=\frac{x_{\rm B}}{2-x_{\rm B}}}$$

$$\bar{u} = 1-u, \quad \bar{v} = 1-v$$

at LO in α_s twist-two amplitudes "factorize"

$$\mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mathcal{Q}^2\right) \stackrel{\mathrm{LO}}{=} \frac{4\alpha_s(\mu_{\mathrm{R}})}{9}\frac{f_V}{\mathcal{Q}} 3\mathcal{I}^V\left(\mu^2\right)\mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mu^2\right)$$

in inverse moment of meson distribution amplitude

$$\mathcal{I}^V(\mu^2) = \frac{1}{3}\int_0^1 du\, \frac{\varphi^V(u,\mu^2)}{u}\,, \qquad \int_0^1 du\, \varphi^V(u,\mu^2) = 1\,,$$

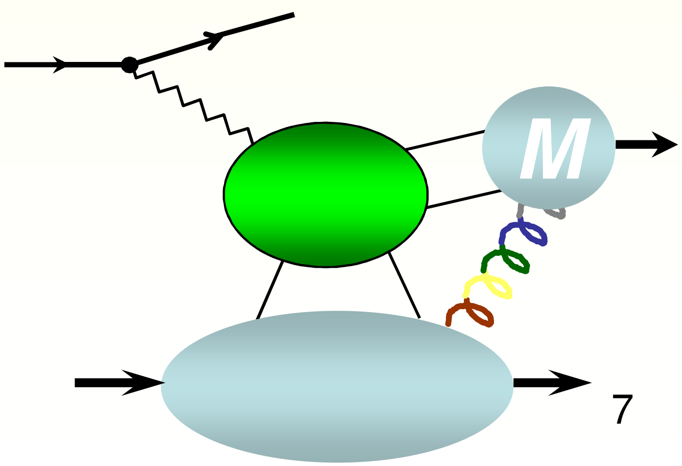
and GPD part "CFFs" , e.g.,

$$\mathcal{H}^{q(+)}(x_{\mathrm{B}}, t, \mu^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^1 dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H^q(x,\xi,t,\mu^2)\Big|_{\xi = x_{\mathrm{B}}/(2-x_{\mathrm{B}})}.$$

factorization might be broken at twist-3 level
due to the initial/final state interaction

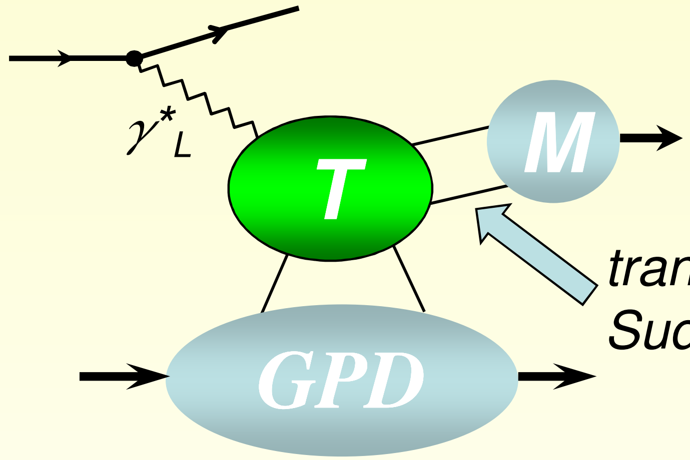
indicated by endpoint singularities at LO

"pragmatic point of view" (introduce a cut-off)
[Mankiewicz et. al (98)]



Other frameworks

GPD inspired model of Goloskokov/Kroll (contains quark + gluons)

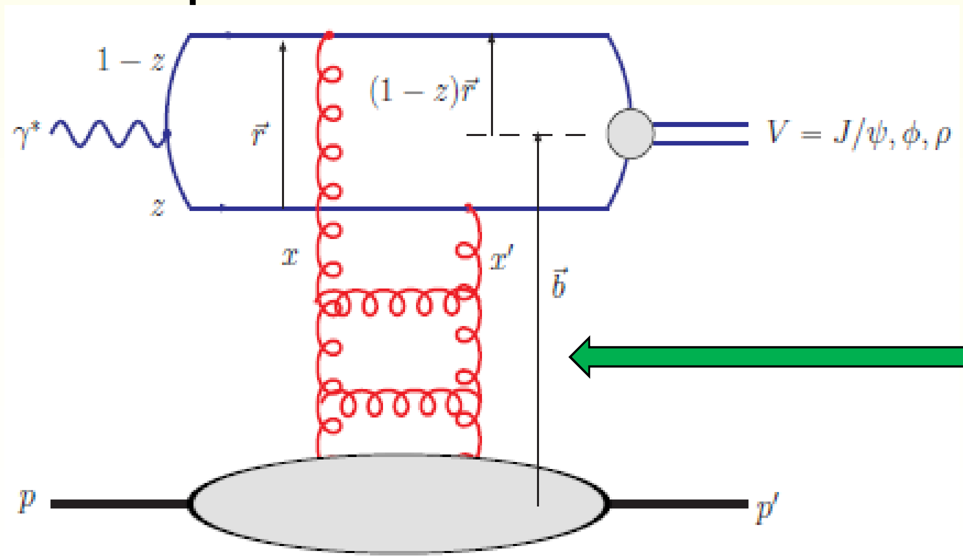


$$\frac{1}{\xi - x - i\epsilon} \quad \Rightarrow \quad \frac{1}{\xi - x + 2\xi\, u\, k_T^2/Q^2 - i\epsilon}$$

transverse degrees
Sudakov suppression

suppressed by energy^2 rather Q^2

color dipole models in various fashions **[A. Mueller (90), ... ,Kowalski et. al, ...]**



applicable at small x, Regge inspired

non-perturbative vector-meson WF

only contains **other** gluons
(**universal color dipole amplitude**)

[picture from Kowalski, Motyka, Watt (06)]

Regge inspired models **[Laget et al]** (also applied at low W)

(longitudinal) DVMP observables

longitudinal TFFs $\mathcal{F}_M(x_B, t, \mathcal{Q}^2)$ for (pseudo)scalar /longitudinal (axial)vector M

$$\epsilon_1^\mu(0)\langle M N|j_\mu|N\rangle = \begin{cases} \overline{u}(p_2,s_2)\left[\not{q}\,\mathcal{H}_M + i\sigma_{\alpha\beta}\frac{q^\alpha\Delta^\beta}{2M_N}\,\mathcal{E}_M\right]u(p_1,s_1) & \text{parity even} \\ \overline{u}(p_2,s_2)\left[\not{q}\gamma_5\,\mathcal{H}_M + \gamma_5\frac{q\cdot\Delta}{2M_N}\,\mathcal{E}_M\right]u(p_1,s_1) & \text{parity odd} \end{cases}$$

longitudinal cross section for polarized (in and out) nucleons

$$\frac{d\sigma^{\gamma_L^* N\to M\,N}}{dt d\varphi} = \frac{2\pi\alpha_{\rm em}}{\mathcal{Q}^4\sqrt{1+\epsilon^2}}\frac{x_B^2}{1-x_B}\frac{1}{2}\Big\{\mathcal{C}_{\rm unp}(\mathcal{F}_M,\mathcal{F}_M^*|s_2) + \Lambda\cos(\theta)\mathcal{C}_{\rm LP}(\mathcal{F}_M,\mathcal{F}_M^*|s_2)$$
$$+\Lambda\cos(\varphi)\sin(\theta)\,\mathcal{C}_{\rm TP+}(\mathcal{F}_M,\mathcal{F}_M^*|s_2) + \Lambda\sin(\varphi)\sin(\theta)\,\mathcal{C}_{\rm TP-}(\mathcal{F}_M,\mathcal{F}_M^*|s_2)\Big\}\,.$$

- polarization transfer measurements allow at least in principle for a complete measurement of the imaginary and real parts of two TFFs
- two observables if polarization of outgoing proton is not measured

$$\mathcal{C}_{\rm unp} = \frac{4(1-x_B)\left(1-x_B\frac{m^2-t}{\mathcal{Q}^2}\right)-\frac{m^2}{M^2}\epsilon^2}{\left(2-x_B-x_B\frac{m^2-t}{\mathcal{Q}^2}\right)^2}\left|\mathcal{H}-\frac{x_B^2\left(1+\frac{m^2-t}{\mathcal{Q}^2}\right)^2+4x_B^2\frac{t}{\mathcal{Q}^2}}{4(1-x_B)\left(1-x_B\frac{m^2-t}{\mathcal{Q}^2}\right)-\frac{m^2}{M^2}\epsilon^2}\mathcal{E}\right|^2 + \frac{1}{(1-x_B)\left(1-x_B\frac{m^2-t}{\mathcal{Q}^2}\right)-\frac{m^2}{4M^2}\epsilon^2}\frac{\widetilde{K}^2}{4M^2}|\mathcal{E}|^2\,,$$

$$\mathcal{C}_{\rm TP-} = -\frac{2}{2-x_B-x_B\frac{m^2-t}{\mathcal{Q}^2}}\frac{\widetilde{K}}{M}\Im{\rm m}\mathcal{H}\mathcal{E}^*\,,$$

partonic decomposition of TFFs

vector mesons
neutral, $J^{PC}=1^{--}$

$\mathcal{F}_M \in \{\mathcal{H}_M, \mathcal{E}_M\}$

$$\mathcal{F}_{\rho^0} = \frac{2}{3\sqrt{2}}\mathcal{F}^{u(+)}_{\rho^0} + \frac{1}{3\sqrt{2}}\mathcal{F}^{d(+)}_{\rho^0} + \frac{1}{\sqrt{2}}\left(\mathcal{F}^{\rm G}_{\rho^0} + \mathcal{F}^{\rm pS}_{\rho^0}\right),$$

$$\mathcal{F}_{\omega} = \frac{2}{3\sqrt{2}}\mathcal{F}^{u(+)}_{\omega} - \frac{1}{3\sqrt{2}}\mathcal{F}^{d(+)}_{\omega} + \frac{1}{3\sqrt{2}}\left(\mathcal{F}^{\rm G}_{\omega} + \mathcal{F}^{\rm pS}_{\omega}\right),$$

$$\mathcal{F}_{\phi} = -\frac{1}{3}\mathcal{F}^{s(+)}_{\phi} - \frac{1}{3}\left(\mathcal{F}^{\rm G}_{\phi} + \mathcal{F}^{\rm pS}_{\phi}\right).$$

charged V-mesons,
via isospin rotation

$$\mathcal{F}_{\rho^+} = \frac{1}{2}\mathcal{F}^{u(-)}_{\rho^0} - \frac{1}{2}\mathcal{F}^{d(-)}_{\rho^0} + \frac{1}{6}\mathcal{F}^{u(+)}_{\rho^0} - \frac{1}{6}\mathcal{F}^{d(+)}_{\rho^0}$$

pseudo scalar mesons
neutral, $J^{PC}=0^{-+}$

$\mathcal{F}_M \in \{\widetilde{\mathcal{H}}_M, \widetilde{\mathcal{E}}_M\}$

$$\mathcal{F}_{\pi^0} = \frac{2}{3\sqrt{2}}\mathcal{F}^{u(-)}_{\pi^0} + \frac{1}{3\sqrt{2}}\mathcal{F}^{d(-)}_{\pi^0},$$

$$\mathcal{F}_{\eta^{(8)}} = \frac{2}{3\sqrt{6}}\mathcal{F}^{u(-)}_{\eta^{(8)}} - \frac{1}{3\sqrt{6}}\mathcal{F}^{d(-)}_{\eta^{(8)}} + \frac{2}{3\sqrt{6}}\mathcal{F}^{s(-)}_{\eta^{(8)}},$$

$$\mathcal{F}_{\eta^{(0)}} = \frac{2}{3\sqrt{3}}\mathcal{F}^{u(-)}_{\eta^{(0)}} - \frac{1}{3\sqrt{3}}\mathcal{F}^{d(-)}_{\eta^{(0)}} - \frac{1}{3\sqrt{3}}\mathcal{F}^{s(-)}_{\eta^{(0)}}.$$

charged PS-mesons,
via isospin rotation

$$\mathcal{F}_{\pi^+} = \frac{1}{2}\mathcal{F}^{u(+)}_{\pi^0} - \frac{1}{2}\mathcal{F}^{d(+)}_{\pi^0} + \frac{1}{6}\mathcal{F}^{u(-)}_{\pi^0} - \frac{1}{6}\mathcal{F}^{d(-)}_{\pi^0}$$

much more channels, eg., *K*, f_0 , *h*, ... mesons

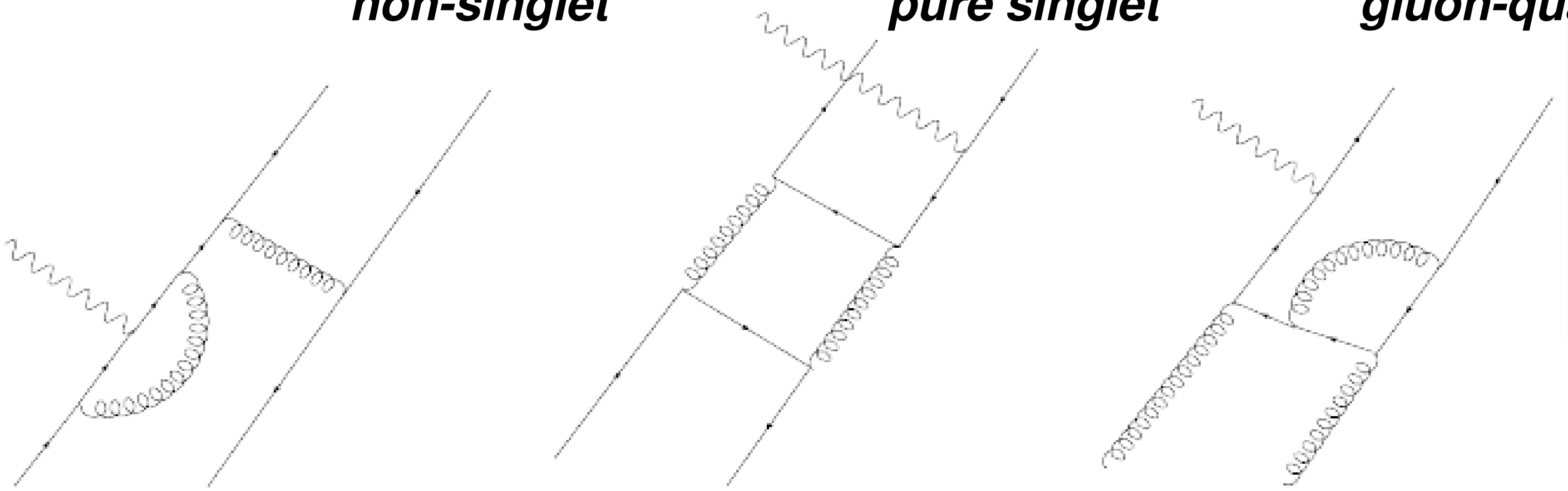
note: hard scattering amplitudes have `some universality'

NLO corrections

• NLO hard scattering coefficients are calculated in momentum fraction space

i. non-singlet quark-quark channel **[Belitsky, DM, 01]** (extend form factor hard scattering part)
ii. pure singlet quark-quark channel **[Ivanov, Szymanowski, Krasnikov 04, KPK et al.]**
iii. gluon-quark channel **[Ivanov, Szymanowski, Krasnikov 04]**

non-singlet ***pure singlet*** ***gluon-quark***



+ many more diagrams

iv. NLO quark-gluon channel is still missing (needed for η/η')

NLO hard scattering part are to be transformed in conformal Mellin-space
(to implement in our flexible GPD model framework)

• NLO evolution kernels in conformal moment space and momentum fraction
[Belitsky, DM, 98] and **[Belitsky, DM, Freund 00]**

NLO example: pure singlet quark-quark channel

momentum fraction representation

factorization log matches evolution ***logarithmical enhancement at endpoints*** ***new pole at u=0***

$$
{}^{\rm pS}T^{(1)}(u,v) = \left[\ln\frac{\mathcal{Q}^2}{\mu_{\rm F}^2} + \frac{\ln\overline{u}}{2} + \ln(v\overline{v}) - 1\right]\frac{\overline{u} - u}{uv\overline{v}}\ln u - \frac{2{\rm Li}_2(u)}{v\overline{v}}
$$

$$
- \left[\frac{1}{2v\overline{v}} + \frac{\ln v}{\overline{v}} + \frac{\ln\overline{v}}{v}\right]\frac{\ln\overline{u}}{u} + \Delta^{\rm pS}T^{(1)}(u,v)\,, \quad \overline{u} \equiv 1-u\,, \;\; \overline{v} \equiv 1-v
$$

mathematically trivial terms (separation in *u* and *v* terms except for *ΔT* addenda)

nontrivial term (free of 1/*(u-v)* singularities) can be nicely written as

$$
\Delta^{\rm pS}T^{(1)}(u,v) = \frac{1}{v\overline{v}}\frac{\partial}{\partial v}v^2\overline{v}\left[\frac{1}{u}\frac{{\rm Li}_2(v) - {\rm Li}_2(u) + \ln\overline{v}\,\ln u - \ln\overline{u}\,\ln u}{u-v}\right]^{\rm sub}
$$

(pole at *u=0* is subtracted)

dispersion relation representation – imaginary part

dispersion relation allows to evaluate the real part from the imaginary one

$$\Re\mathrm{e}\mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mathcal{Q}^2\right) = \frac{1}{\pi}\mathrm{PV}\int_0^1 dx\,\frac{2x}{x^2-\xi^2}\Im\mathrm{m}\mathcal{H}^{pV}\left(\frac{2x}{1+x}, t, \mathcal{Q}^2\right) + \mathcal{C}\left(t, \mathcal{Q}^2\right)|_{\xi=\frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}}$$

only GPD in the outer region is needed

“D-term” projection is needed to calculate the subtraction constant

imaginary part can be straightforwardly evaluated *($x > \xi$)*

$$\Im\mathrm{m}\mathcal{F}(x_{\mathrm{B}}, t, \mathcal{Q}^2) = \frac{C_F f_{\mathrm{M}}}{N_c \mathcal{Q}} F(x, \xi, t, \mu_{\mathrm{F}}^2)\int_\xi^1 \frac{dx}{x}\, t\left(\frac{x}{\xi}, v\middle|\cdots\right) \overset{v}{\otimes} \varphi(v, \mu_\varphi^2)\Big|_{\xi=\frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}}$$

u=0 pole turns into r=0 pole

$$\begin{aligned}
{}^{\mathrm{pS}}t^{(1)}(r, v|\cdots) &= \left[\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2} + \ln\frac{1-r}{1+r} + \ln(v\bar{v}) - 1\right]\frac{1}{r(1+r)v\bar{v}} - \frac{\ln\frac{1+r}{2r}}{(1+r)v\bar{v}} \\
&\quad + \left[\frac{1}{2v\bar{v}} + \frac{\ln v}{\bar{v}} + \frac{\ln\bar{v}}{v}\right]\frac{1}{1+r} + \Delta^{\mathrm{pS}}t^{(1)}(r, v)\,, \\
\Delta^{\mathrm{pS}}t^{(1)}(r, v) &= \frac{1}{v\bar{v}}\frac{\partial}{\partial v} v\bar{v}\left[\frac{2rv}{1+r}\,\frac{\ln\frac{1+r}{2rv}}{1+r-2rv}\right].
\end{aligned}$$

reminder conformal representation of distribution amplitudes

conformal partial wave expansion of meson distribution amplitude

$$\varphi(u,\mu^2) = \sum_{\substack{k=0 \\ \text{even}}}^{\infty} 6u\bar{u}\, C_k^{3/2}(u-\bar{u})\, \varphi_k(\mu^2)\,, \quad \varphi_0 = 1\,, \quad \bar{u} = 1-u$$

conformal partial wave amplitudes evolve autonomously at LO

$$\int_0^1 du\, T(u)\varphi(u) \quad \Rightarrow \quad \sum_{\substack{k=0 \\ \text{even}}} T_k \varphi_k \qquad T_k = \int_0^1 du\, T(u)\, 6u\bar{u}\, C_k^{3/2}(u-\bar{u})$$

to calculate T_k one might use Rodrigues formula + partial integration

$$T_k = \frac{3(2+k)}{k!} \int_0^1 du\, (u\bar{u})^{k+1} \frac{d^k}{du^k} T(u)$$

simple LO example: $T(u) = \frac{1}{u}$ $\qquad T_k = 3(2+k)(-1)^k \int_0^1 du\, u^{k+1} = 3(-1)^k$

all NLO expressions (LO decorated by *ln* and Li_2 functions) are analytically known

fixing $(-1)^k = \sigma = \mp 1$ allows to employ Carlson theorem to find expressions for complex k
(analytic continuation of a function with integer argument)

conformal representation of transition form factors

$$ {}^{\mathrm{pS}}T^{(1)}(u,v) \quad \Rightarrow \quad {}^{\mathrm{pS}}T^{(1)}_{jk} \propto \int_0^1 du \int_0^1 dv\, u\bar{u}\, C_j^{3/2}(u-\bar{u})\, {}^{\mathrm{pS}}T^{(1)}(u,v) v\bar{v}\, C_k^{3/2}(v-\bar{v}) $$

NLO hard scattering part posses the form

$T = \Sigma_{f,g}\, f(u)\, g(v) + \Delta T(u,v)$ $\Longrightarrow$ $T_{jk} = \Sigma_{f,g}\, f_j\, g_k + \Delta T_{jk}$

known find appropriate representation

use "double dispersion relation" to get moments of ΔT_{jk} *for complex j (k)*

$$ \Delta^{\mathrm{pS}}T^{(1,F)}(u,v) = \int_0^1 dy \int_0^1 dz\, \frac{1}{1-uy}\, \frac{1}{y+z-yz} \left[\frac{y}{z} - 1 - z\,\frac{\vec{d}}{dz}\right] \frac{z}{1-\bar{v}z} $$

conformal moments of *1/(1-uy)* are easily calculable and have the needed analytic properties ($\nu \in \{3/2, 5/2\}$)

$$ \widetilde{p}_k^{(\nu)}(y) \propto \int_0^1 du\, \frac{1}{1-uy}\, (u\bar{u})^{\nu-1/2}\, C_k^{\nu}(u-\bar{u}) \Rightarrow \widetilde{p}_k^{(3/2)}(y) \propto y^k \int_0^1 du\, \frac{(u\bar{u})^{k+1}}{(1-uy)^{k+1}} $$

analytical continuation of ΔT_{jk} can be numerical performed

$$ \Delta^{\mathrm{pS}}T^{(1)}_{jk} = \int_0^1 dy \int_0^1 dz\, \widetilde{p}_j^{(3/2)}(y) \left[\frac{y}{z} - 1 - z\,\frac{\vec{d}}{dz}\right] z\widetilde{p}_k^{(3/2)}(z) $$

conformal moments (pure singlet)

$$ {}^{\mathrm{pS}}T^{(1)}_{j,k}(Q^2/\mu_F^2) \quad = \quad T^{(0)}_{j,k}\, {}^{\mathrm{pS}}c^{(1)}_{j,k}\left(\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2}\right) $$

$$ {}^{\mathrm{pS}}c^{(1)}_{j,k} \;=\; \left[-\ln\frac{\mathcal{Q}^2}{\mu_{\mathrm{F}}^2} + 2S_1(j+1) + 2S_1(k+1) - 1\right] \frac{{}^{\mathrm{G}\Sigma}\gamma_j^{(0,\mathrm{F})}}{j+3} - \left[\frac{1}{2} + \frac{1}{(j+1)(j+2)} + \frac{1}{(k+1)(k+2)}\right] \frac{2}{(j+1)(j+2)} + \Delta^{\mathrm{pS}}c^{(1)}_{j,k}\,, $$

j=0 pole is contained in anomalous dimension

$$ {}^{\mathrm{G}\Sigma}\gamma_j^{(0,\mathrm{F})} = -2\,\frac{(j+1)(j+2)+2}{j(j+1)(j+2)} $$

- analog structure in NS and gluon channels
- all Δc_{jk} are analytically evaluated, e.g.,
  (they are harmless at j=0, large j or large k)

$$ \Delta^{\mathrm{pS}}c^{(1)}_{jk} \;=\; \frac{(k)_4\left[\Delta S_2\left(\frac{j+1}{2},\frac{k}{2}\right) - \Delta S_2\left(\frac{j+1}{2},\frac{k+2}{2}\right)\right]}{2(2k+3)}, $$

$$ \Delta S_2\left(\frac{j+1}{2},\frac{k+1}{2}\right) \;=\; \frac{S_2\left(\frac{j+1}{2}\right) - S_2\left(\frac{j}{2}\right) - S_2\left(\frac{k+1}{2}\right) + S_2\left(\frac{k}{2}\right)}{2(j-k)(j+k+3)}, $$

Myths

1. myth: canonical Q^2-scaling, derived from dimension counting rules

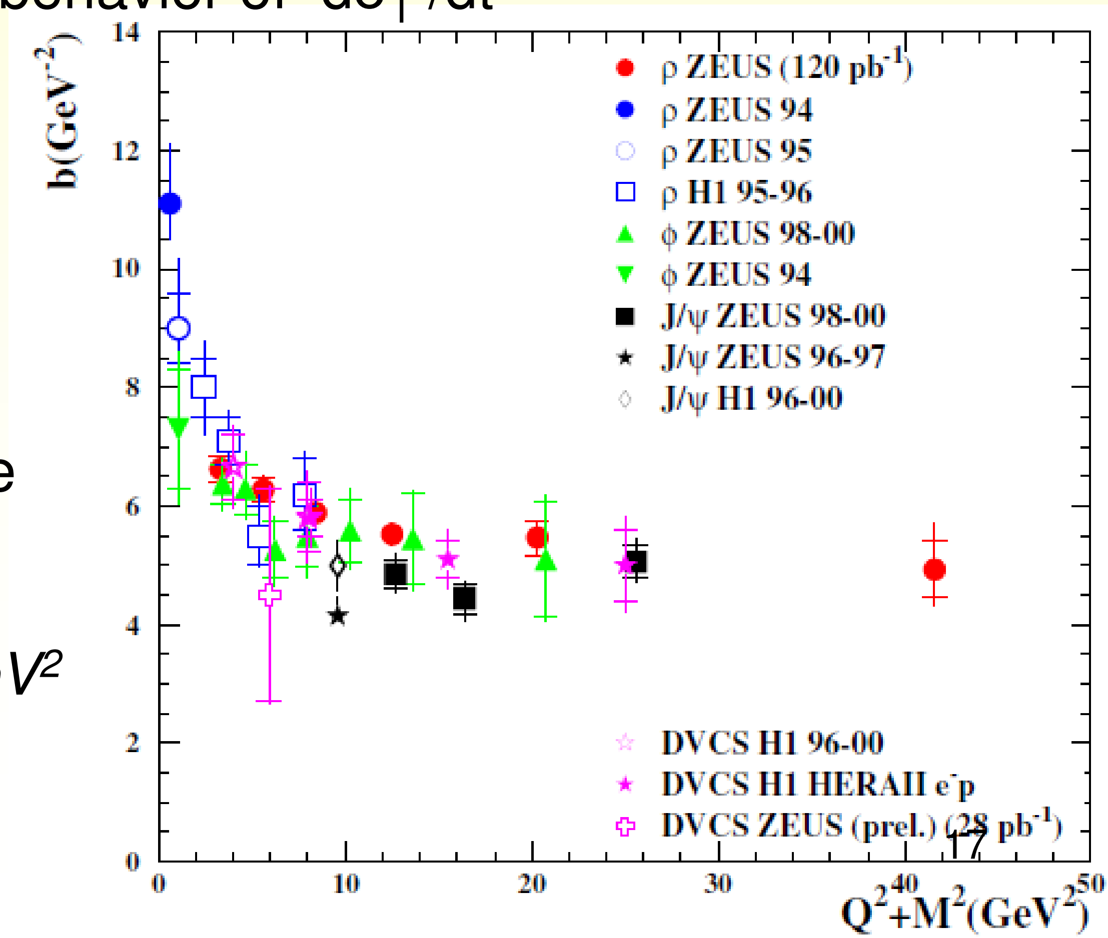
$$\frac{d\sigma^{\gamma^*_L p \to Mp}}{dt} \propto \frac{1}{Q^6} \quad \text{and} \quad \frac{d\sigma^{\gamma^*_T p \to Mp}}{dt} \propto \frac{1}{Q^8} \quad \text{for fixed } x_B$$

- scaling violations are predicted by pQCD
- not to much care is needed on the behavior of $d\sigma_T/dt$

2. myth: in a hard process the t-slope does not evolve with Q^2

- quark species and gluons might have different t-slopes, which implies Q^2-evolution of t-slope

3. myth: onset of hard regime for H1/ZEUS data is at Q^2 ~ $15\ GeV^2$



Size of NLO corrections

[Belitsky, DM 01] large NLO corrections in the NS sector (proportional to C_F, β_0)

known from pion form factor:
- removing β_0 term will push $\alpha_s(\mu_R)$ into non-perturbative region
- C_F proportional corrections indicate Sudakov suppression (different sign)

discussion how to set Brodsky, Lepage, Mackenzie scale to eliminate β_0 proportional hard scattering coefficient (dispersion relation) **[Pire et al., Brodsky et al.]**

[Ivanov, Szymanowski, Krasnikov 04] big NLO corrections at small *x*

[Diehl, Kugler 07] model studies without NLO evolution

analytic expressions allows easily to understand nature of NLO corrections
- big at small *x* (*j=0* pole) and large at large *x* (large *j*)
- increase with growing *k*
- still a large scale dependence at NLO

➢ NLO corrections are model dependent (from huge to moderate)

pragmatic point of view:

- not much hope to understand power suppressed corrections
- work hard to evaluate radiative corrections (? resummation)
- explore how the collinear factorization approach works to describe data

First step DVCS/DVMP-fits to H1/ZEUS @LO/NLO

DVCS cross section
$$\frac{d\sigma^{\gamma^* p\to\gamma p}}{dt} \overset{\mathrm{Tw-2}}{\approx} \pi\alpha^2 \frac{x_{\mathrm{B}}^2}{\mathcal{Q}^4} \left|\mathcal{H}\left(x_{\mathrm{B}}, t, \mathcal{Q}^2\right)\right|^2 + \cdots$$

DVMP cross sections
$$\frac{d\sigma^{\gamma_{\mathrm{L}}^* p\to V_{\mathrm{L}} p}}{dt} \overset{\mathrm{Tw-2}}{\approx} 4\pi^2\alpha \,\frac{x_{\mathrm{B}}^2}{\mathcal{Q}^4} \left|\mathcal{H}_V\left(x_{\mathrm{B}}, t, \mathcal{Q}^2\right)\right|^2 + \cdots$$

$$\mathcal{H}_V\left(x_{\mathrm{B}}, t, \mathcal{Q}^2\right) \overset{\mathrm{LO}}{=} \frac{4\alpha_s(\mu_{\mathrm{R}})}{9}\frac{f_V}{\mathcal{Q}} 3\mathcal{I}^V\left(\mu^2\right)\mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mu^2\right)$$

$$\mathcal{I}^V(\mu^2) = \frac{1}{3}\int_0^1 du\, \frac{\varphi^V(u,\mu^2)}{u}\,, \qquad \int_0^1 du\,\varphi^V(u,\mu^2) = 1\,,$$

$$\mathcal{H}^{p\rho^0} \overset{\mathrm{LO}}{=} \frac{1}{\sqrt{2}}\left(\frac{2}{3}\mathcal{H}^{u(+)} + \frac{1}{3}\mathcal{H}^{d(+)} + \frac{3}{4}\mathcal{H}^G\right)$$

$$\mathcal{H}^{p\omega} \overset{\mathrm{LO}}{=} \frac{1}{\sqrt{2}}\left(\frac{2}{3}\mathcal{H}^{u(+)} - \frac{1}{3}\mathcal{H}^{d(+)} + \frac{1}{4}\mathcal{H}^G\right)$$

$$\mathcal{H}^{p\phi} \overset{\mathrm{LO}}{=} (-1)\left(\frac{1}{3}\mathcal{H}^{s(+)} + \frac{1}{4}\mathcal{H}^G\right)$$

$$\mathcal{H}^{q(+)}(x_{\mathrm{B}}, t, \mu^2) \overset{\mathrm{LO}}{=} \int_{-1}^1 dx\,\left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H^q(x,\xi,t,\mu^2)\Big|_{\xi = x_{\mathrm{B}}/(2-x_{\mathrm{B}})}\,.$$

$$\mathcal{H}^G(x_{\mathrm{B}}, t, \mu^2) \overset{\mathrm{LO}}{=} \int_{-1}^1 dx\,\frac{1}{2x}\left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H^G(x,\xi,t,\mu^2)\Big|_{\xi = x_{\mathrm{B}}/(2-x_{\mathrm{B}})}$$

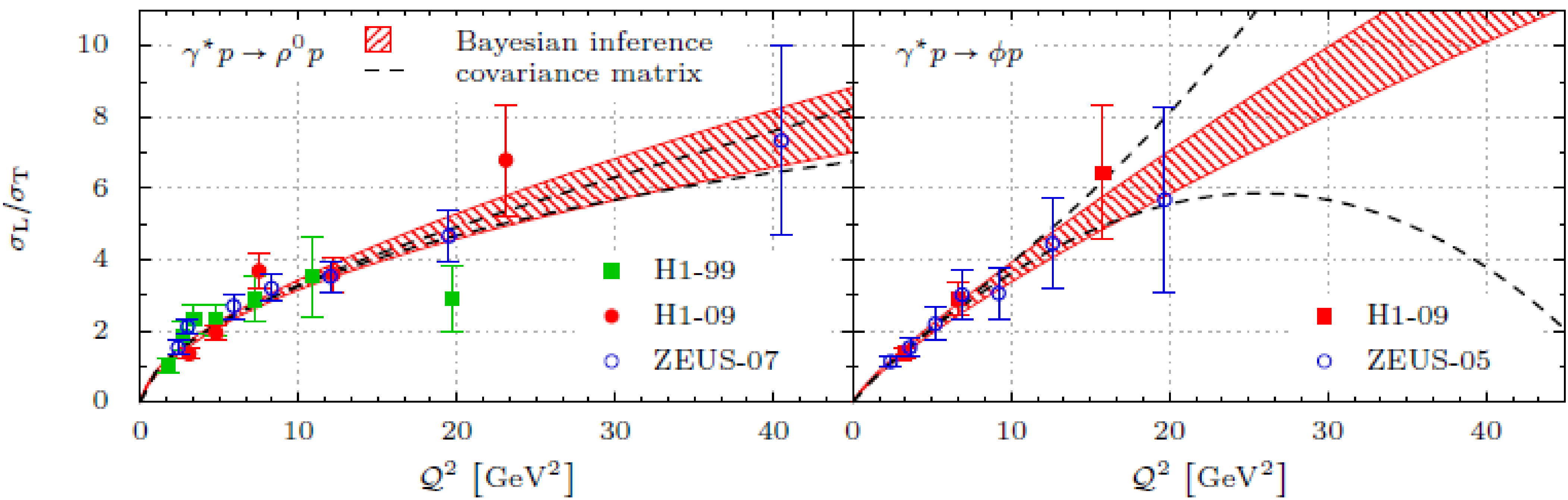
Cross sections and R-ratio

most of H1/ZEUS measurements are given for integrated cross section

$$\sigma(W,Q^2) = \left[\varepsilon(W,Q^2) + \frac{1}{R^{\exp}(Q^2)}\right] \int_{|t_{\min}|}^{|t_{\rm cut}|} dt\, \frac{d\sigma_{\rm L}(x_{\rm B},t,Q^2)}{dt} \qquad |t_{\rm cut}| < Q^2$$

R-ratio is extracted via s-channel helicity conservation hypothesis
it is assumed to be independent on *W* and *t* *(this is not entirely true)*
We use the model:

$$R^{\exp}(Q^2) = \frac{Q^2/m_V^2}{(1+aQ^2/m_V^2)^p} \quad \text{with} \quad \left\{\begin{array}{l} a=2.2\,, \quad p=0.451 \\ a=25.4\,, \ p=0.180 \end{array}\right\} \ \text{for} \ \left\{\begin{array}{c} \rho^0 \\ \phi \end{array}\right\}$$



Bayesian inference estimation

posterior probability distribution function (pdf) is calculated from Bayes theorem

$$f(\mathbf{p}|DI) = \frac{f(\mathbf{p}|I)\,P(D|\mathbf{p}I)}{P(D|I)}$$

D data
p set of parameters
I assumptions

*f(**p**|I)* prior pdf is given
*P(D|**p**I)* likelihood function has to be calculated
P(D|I) normalization has to be calculated

once posterior pdf is know all other quantities can be calculated
e.g., the pdf of a quantity *Q(**p**)* reads

$$f(q|DI) = \int d\mathbf{p}\,\delta(q - Q(\mathbf{p}))\,f(\mathbf{p}|DI)$$

the mean and variance of a parameter reads

$$\mathrm{E}(p_i) = \int d\mathbf{p}\,p_i\,f(\mathbf{p}|DI) \quad \text{and} \quad \mathrm{Var}(p_i) = \mathrm{E}(p_i^2) - \mathrm{E}^2(p_i)$$

advantages: simple (and correct) error propagation
judging on parameter sensitivity
comparing different models or approximations
disadvantages: computational power is needed, time consuming

GPD ansatz from t-channel view

- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*
- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is

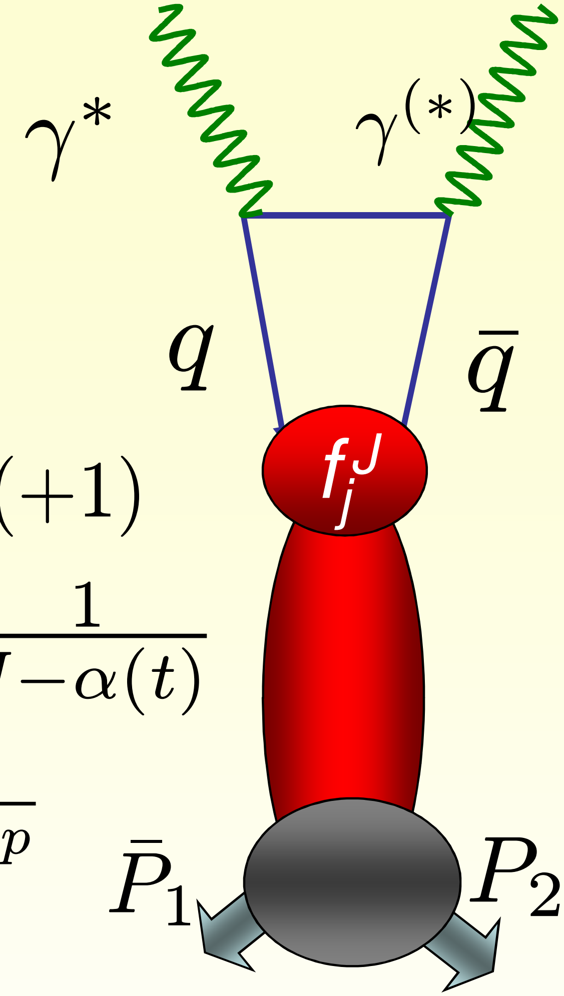
$$f_j^J\,,\; J \leq j\,(+1)$$

- mesons propagate with

$$\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$$

- decaying into nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***

$$\frac{1}{(1-\frac{t}{M^2(J)})^p}$$



➢ (conformal) GPD moments expanded in Wigner`s rotation matrices

$$F_j(t,\eta) = \sum_J^{j\,(+1)} \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}\, \eta^{j\,(+1)-J}\, \hat{d}_J^F(\eta)\,, \quad \hat{d}_J^F(\eta=0)=1$$

- labeling by *t*-channel quantum numbers J^{PC} **[Lebed, Ji (00),...]**
- so-called D-term arises from *0*$^{++}$*, (f*^{0} *or σ) 2*$^{++}$*, 4*$^{++}$*, ...,* has even *J=j+1 (or j = -1 in DR)* pole (*J (=0)* has multiple meanings **[KMP-K(07&08)]**)
- usable for large *x* (employing effective rotation matrices)

Least square fits @LO

flexible GPD-parameterization 2x6-1 parameters , fixed meson DAs

$$\boldsymbol{p} = \boldsymbol{p}^{\mathrm{sea}} \cup \boldsymbol{p}^{\mathrm{G}} \quad \text{with} \quad \boldsymbol{p}^{p} = \{N^{p}, \alpha^{p}, \alpha'^{\,p}, M^{p}, s_2^{p}, s_4^{p}\}$$

momentum fraction sum rule is implemented as

$$N^{\mathrm{sea}} + N^{\mathrm{val}} + N^{\mathrm{G}} = 1 \quad \text{with} \quad N^{\mathrm{val}} = 0.4$$

DIS: PDFs are sensitive only to 2x2-1 parameters: $\{N^{\mathrm{sea}}, \alpha^{\mathrm{sea}}, \alpha^{\mathrm{G}}\}$

DVCS/DVMP: additional 2x4 GPD parameters are available to control

t-dependency is controlled by

$$\{\alpha'^{\,\mathrm{sea}} \sim 0.15\,\mathrm{GeV}^{-2}, \alpha'^{\,\mathrm{G}} \sim 0.15\,\mathrm{GeV}^{-2}, M^{\mathrm{sea}} \sim \sqrt{0.6}\mathrm{GeV}, M^{\mathrm{G}} \sim \sqrt{0.7}\mathrm{GeV}\}$$

normalization (including its Q^2 dependence) is controlled by four parameters

$$\{s_2^{\mathrm{sea}}, s_4^{\mathrm{sea}}, s_2^{\mathrm{G}}, s_4^{\mathrm{G}}\}$$

NOTE: Shuvaev et al. misunderstanding $s_2^{\mathrm{sea}} = s_4^{\mathrm{sea}} = s_2^{\mathrm{G}} = s_4^{\mathrm{G}} = 0$

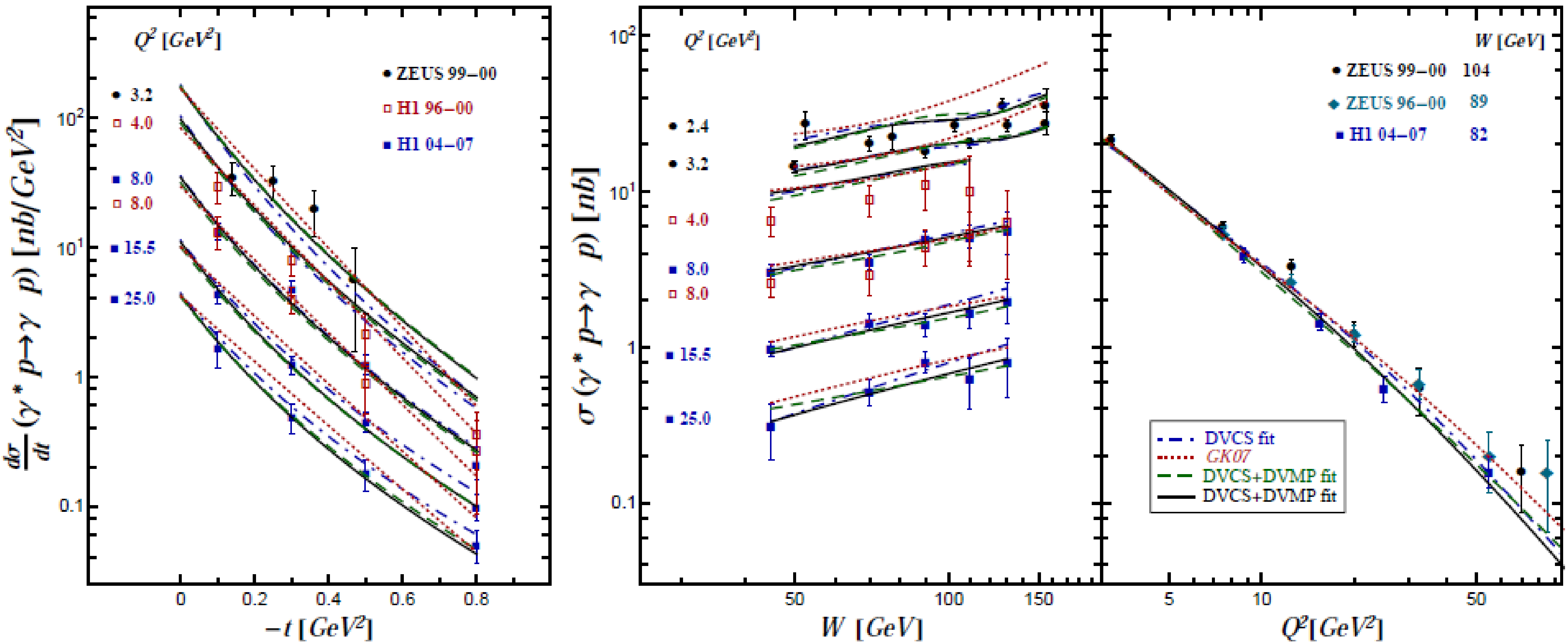
RDDA models are mimicked by $s_4^{\mathrm{sea}} = s_4^{\mathrm{G}} = 0$

DVCS+DVMP fit to H1/ZEUS data

strategies: pure DVCS fit χ^2/d.o.f. = 130/(126-3)
DVCS + H1 DVMP fit χ^2/d.o.f. = 342/(230-6)
DVCS + H1/ZEUS DVMP fit -very soft gluon χ^2/d.o.f. = 618/(304-7)
confronting GK07 model with DVCS (χ^2/n.o.p. = 226/126)

R and normalization errors are not taken into account, cut $Q^2 > 4\ \mathrm{GeV}^2$ for DVMP data

DVCS data dominated by quark GPD
gluon GPD is to some extend not pinned down

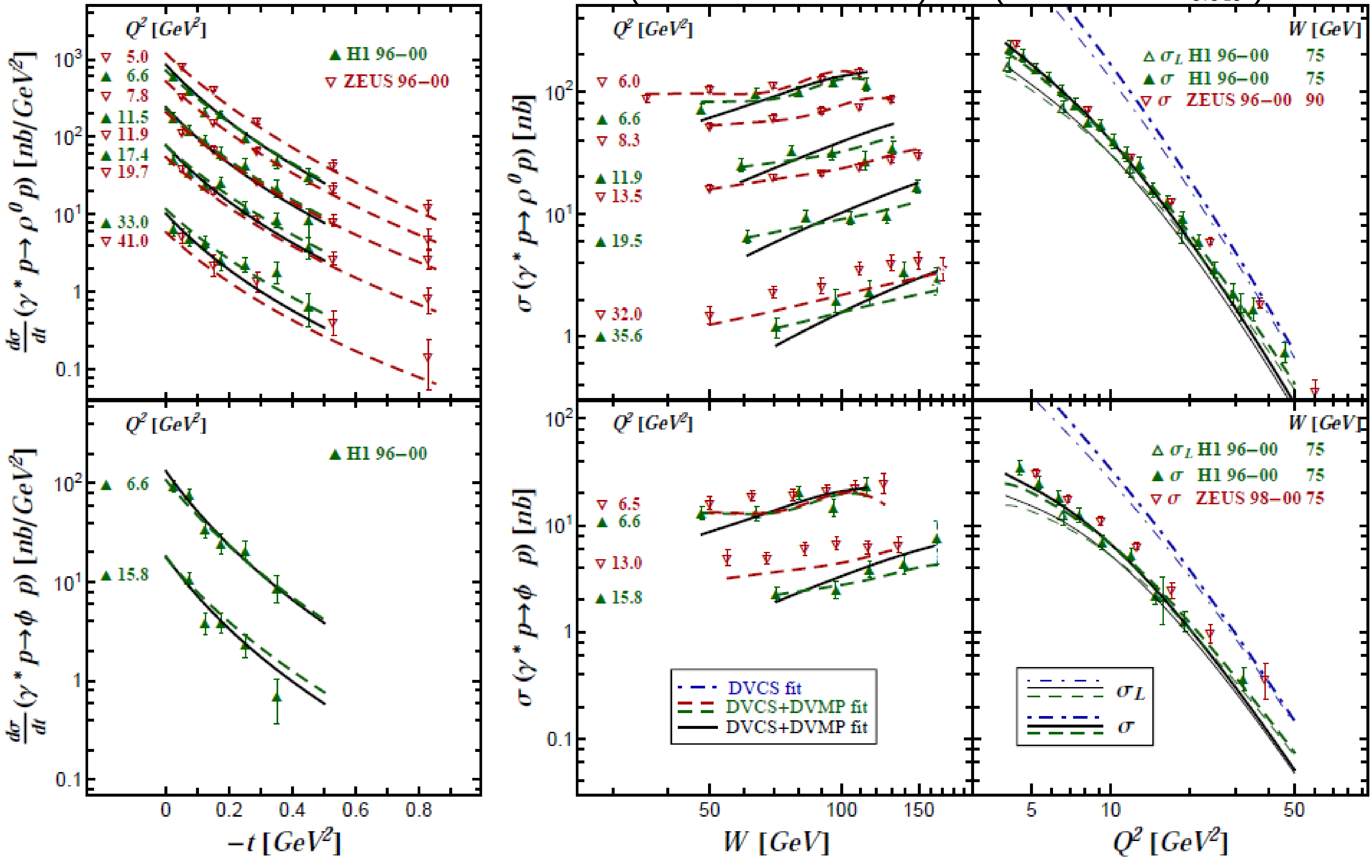


H1 and ZEUS DVMP data

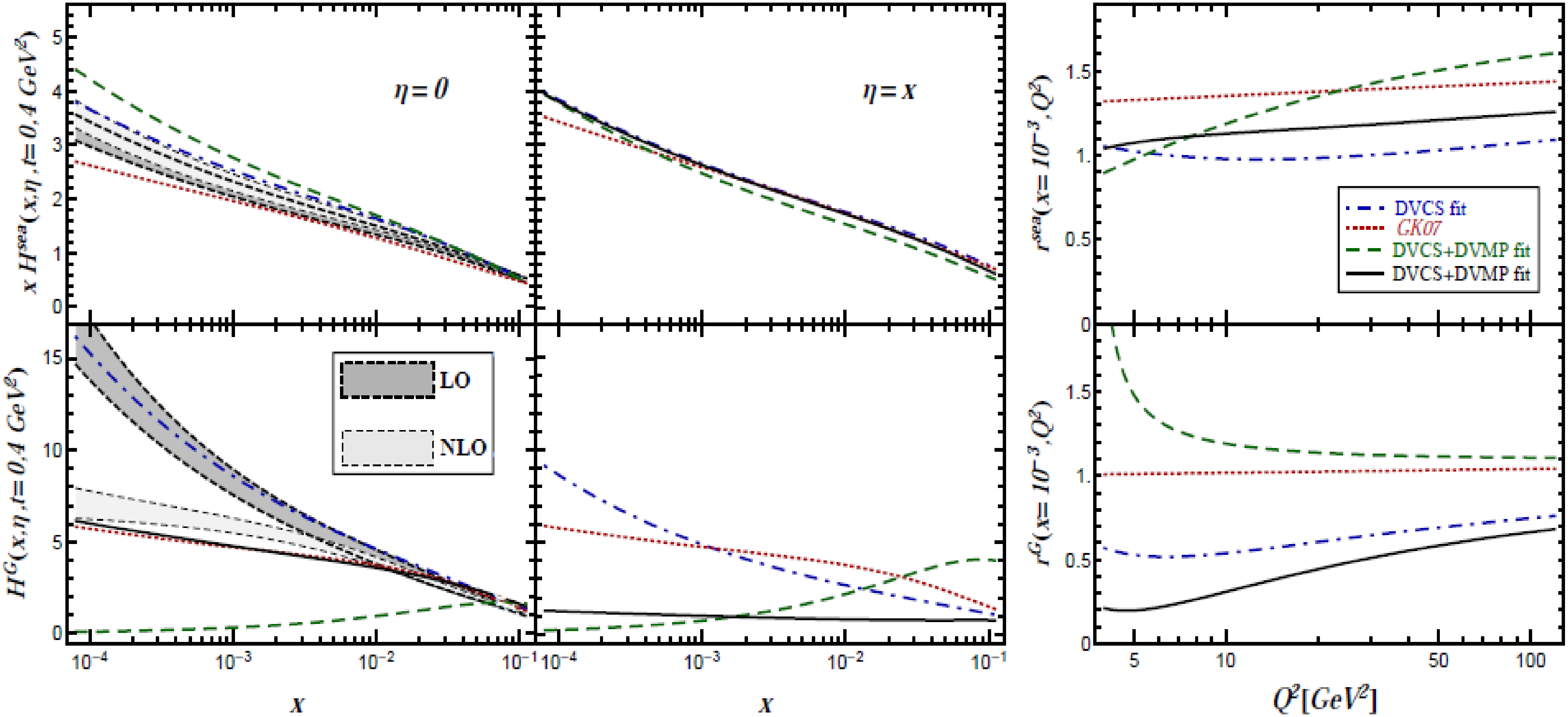
are compatible
however, there are differences

$$\left\{\begin{matrix} b^{\rm H1}(\mathcal{Q}^2 = 11.5\,{\rm GeV}^2) \\ b^{\rm ZEUS}(\mathcal{Q}^2 = 11\,{\rm GeV}^2) \end{matrix}\right\} = \left\{\begin{matrix} 6.72 \pm 0.53 \,{}^{+0.23}_{-0.25} \\ 5.7 \;\; \pm 0.5 \;\; {}^{+0.2}_{-0.2} \end{matrix}\right\} /{\rm GeV}^2 ,$$

$$\left\{\begin{matrix} \delta^{\rm H1}(\mathcal{Q}^2 = 6.6\,{\rm GeV}^2) \\ \delta^{\rm ZEUS}(\mathcal{Q}^2 = 6\,{\rm GeV}^2) \end{matrix}\right\} = \left\{\begin{matrix} 0.57 \pm 0.10 \;\; {}^{+0.05}_{-0.07} \\ 0.4 \;\; \pm 0.052 \,{}^{+0.048}_{-0.045} \end{matrix}\right\} ,$$



partonic interpretation



r^{sea}~*1* (small skewness effect at LO) of sea quarks are driven by DVCS data

r^G *< 1* gluon GPD is suppressed at LO

very soft gluon GPD is disfavored by DIS fit

GK07 model is based on NLO PDFs [(very) good DVCS description at LO]
interchange of skewing and evolution provides a very desired GPD behavior
remember Freund/McDermott could not reach DVCS description @LO with similar model

Bayesian inference estimates @LO/NLO

to have a better LO description we over-flexible the GPD 2x10-1 parameters

can be also used to study *η*-*t* correlation (important issue for tomography)

$$\boldsymbol{p} = \boldsymbol{p}^{\rm sea} \cup \boldsymbol{p}^{\rm G} \quad \text{with} \quad \boldsymbol{p}^{p} = \{N^p, \alpha^p, \alpha'^{\,p}, M^p, s_2^p, \alpha_2^p, M_2^p, s_4^p, \alpha_4^p, M_4^p\}$$

momentum fraction sum rule reads now

$$N^{\rm sea} + N^{\rm val} + N^{\rm G} = 1 \quad \text{with} \quad N^{\rm val} = 0.4 \pm 0.05$$

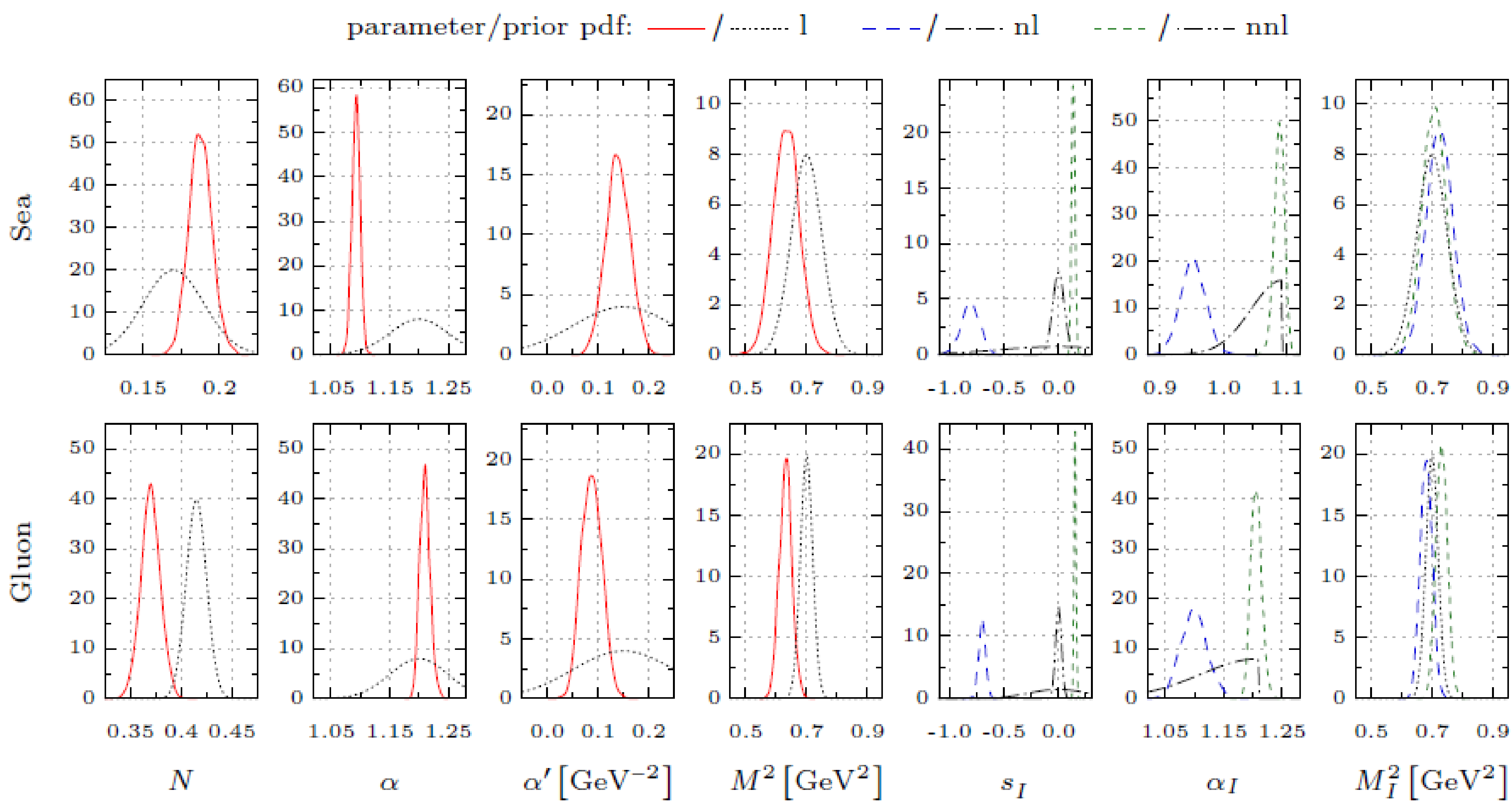
additional 8 parameters $\{\alpha_2^{\rm sea}, M_2^{\rm sea}, \alpha_4^{\rm sea}, M_4^{\rm sea}, \alpha_2^{\rm G}, M_2^{\rm G}, \alpha_4^{\rm G}, M_4^{\rm G}\}$
control *x*- and *t*-dependence of higher SO(3)-PWs

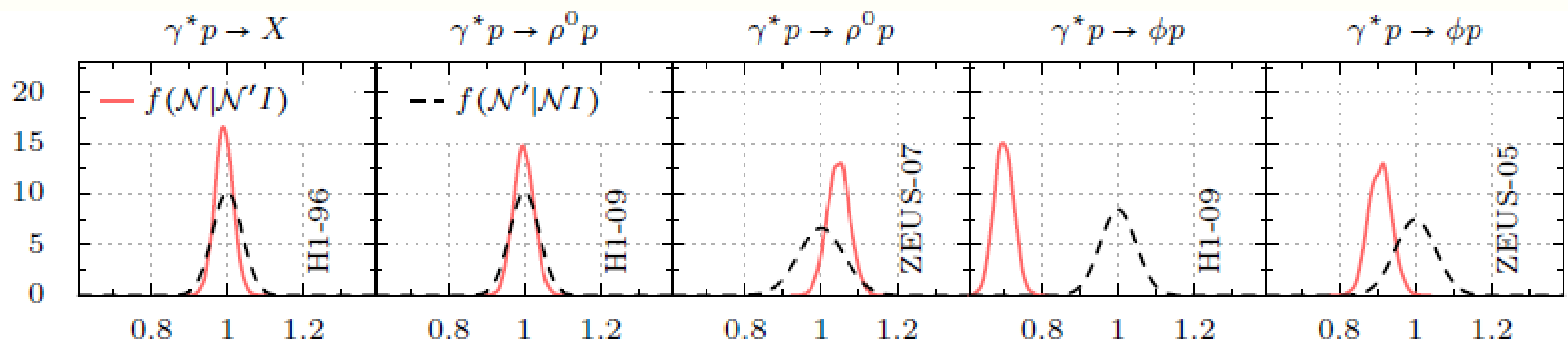
normalization errors of cross section measurements are taken now into account

two strategies are explored: **[Tobias Lautenschlager]**

2x10-1 GPD parameters with fixed meson DAs: favors NLO, works at LO

2x6-1 GPD parameters + 2 DA parameters: works at NLO

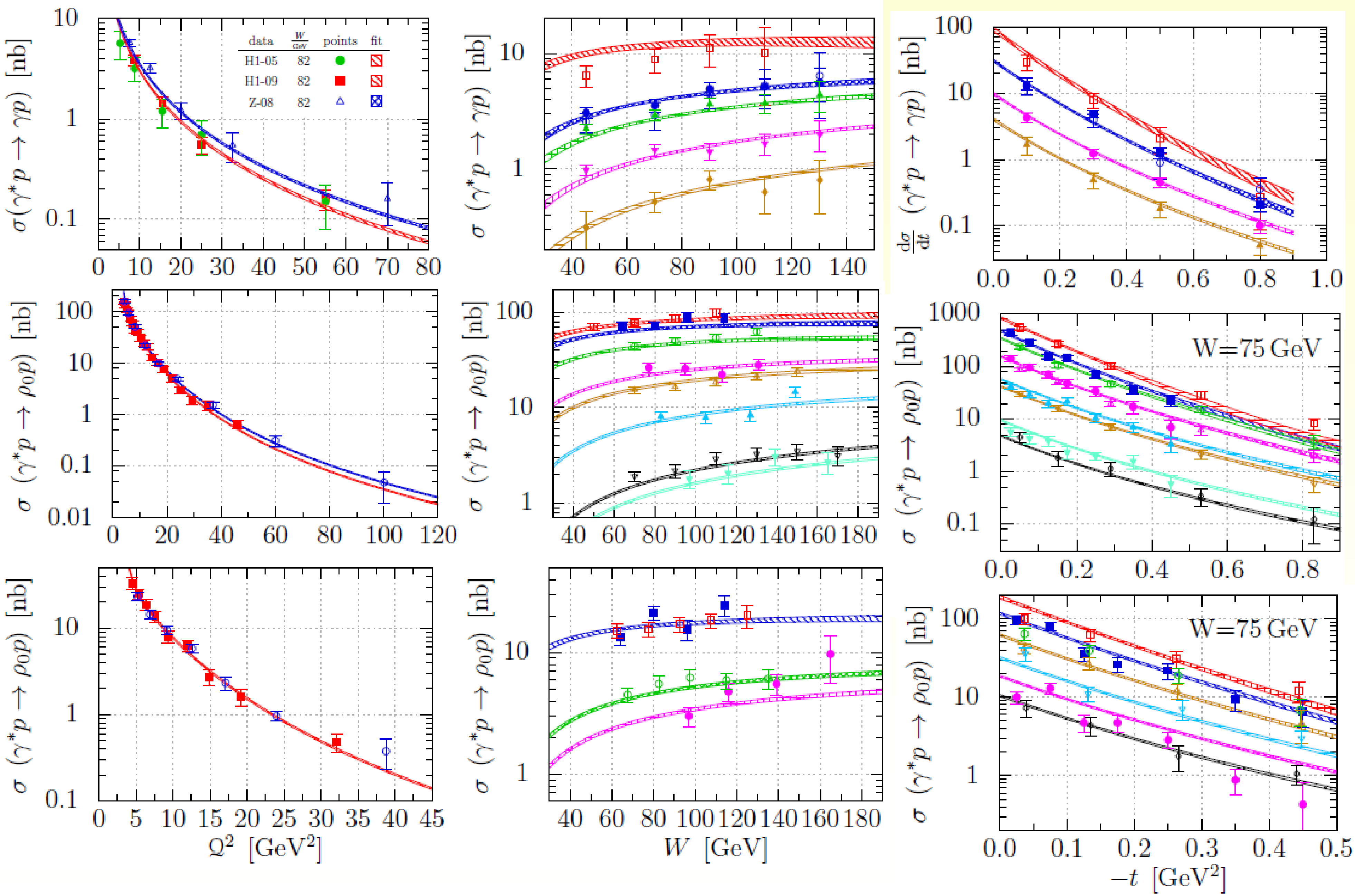

parameter/prior pdf: l nl nnl
Sea
Gluon
N
α
α′ [GeV^-2]
M^2 [GeV^2]
s_I
α_I
M_I^2 [GeV^2]


γ*p → X
γ*p → ρ^0 p
γ*p → ρ^0 p
γ*p → φp
γ*p → φp
f(N|N′I)
f(N′|NI)
H1-96
H1-09
ZEUS-07
H1-09
ZEUS-05

DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (soon)]**

works at NLO ($Q^2 > 4$ GeV^2), done with Bayes theorem (probability distribution function)

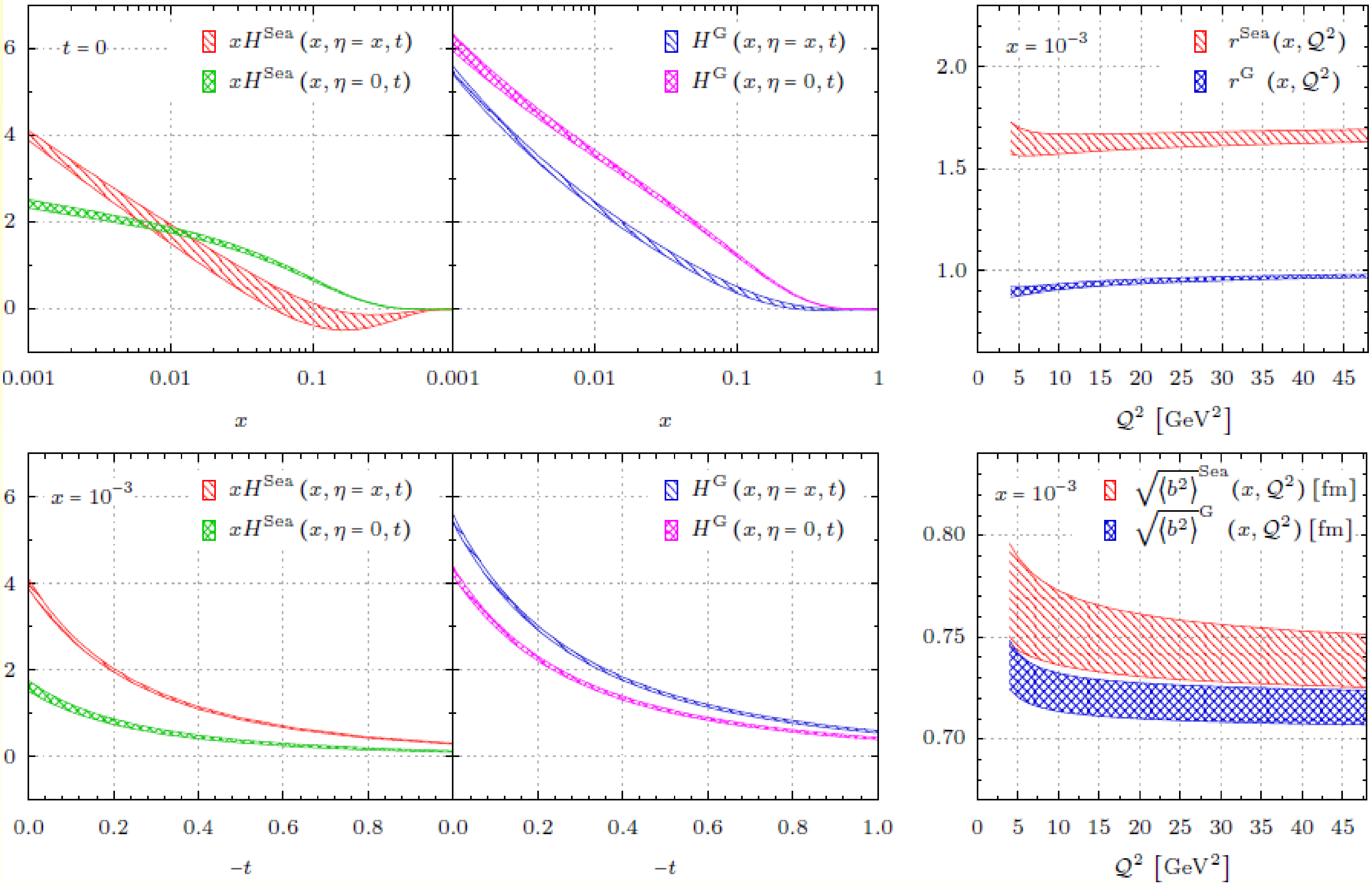


fixed:

meson DA flavor content

errors might be perhaps larger

entirely model dependency for x> 10^{-2}



- going from LO to NLO increases the skewness ratios (known since \`ever', **[KMP-K(07)]**)
- gluons are more centralized as sea quarks (expected from DVCS & *J/ψ* interpretation)
- cross-talk of skewness and *t*-dependency has been addressed by pdf
- NLO GPDs look rather compatible to Goloskokov/Kroll and Martin et. al finding
- there is also DVCS beam charge and perhaps **beam spin** data are coming up

Conclusions

❖ NLO fit to DVCS and DVMP works at small x_B for $Q^2 > 4\ GeV^2$

❖ contradicts common myths/interpretation

•

- exclusive physics in H1/ZEUS kinematics is dominated by gluons
- onset of perturbation regime is at $\sim 15\ GeV^2$ or so

❖ GPD interpretation for $Q^2 > 4\ GeV^2$ states that at LO

- quark exchanges at small x_B are more important as thought
- gluons in off-forward kinematics are suppressed

❖ GPD interpretation for $Q^2 > 4\ GeV^2$ states that at NLO

- GPDs are compatible with Shuvaev`s LO ratios
- gluons are more centralized as sea quarks

❖ another partonic GPD interpretation arises in GK framework
(DVMP handbag + transverse degrees of freedom)